HL2 Summer CALC OK Assignment *[54 marks]*

On 1st January 2020, Laurie invests $\$P$ in an account that pays a nominal annual interest rate of 5.5 %, compounded **quarterly**.

The amount of money in Laurie's account **at the end of each year** follows a geometric sequence with common ratio, r.

1a. Find the value of r, giving your answer to four significant figures. *[3 marks]*

1b. Laurie makes no further deposits to or withdrawals from the account. *[3 marks]*

Find the year in which the amount of money in Laurie's account will become double the amount she invested.

Consider $\begin{pmatrix} 11 \\ a \end{pmatrix} = \frac{11!}{a!\,9!}$.

2a. Find the value of a. *[2 marks]*

2b. Hence or otherwise find the coefficient of the term in x^9 in the expansion *[4 marks]* of $(x+3)^{11}$.

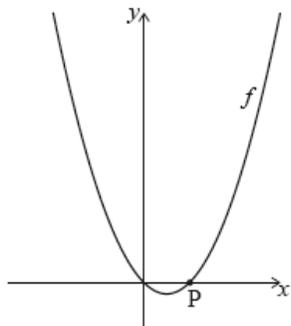The first two terms of a geometric sequence are $u_1 = 2.1$ and $u_2 = 2.226$.

3a. Find the value of r. *[2 marks]*

3b. Find the value of u_{10}. *[2 marks]*

3c. Find the least value of n such that $S_n > 5543$. *[3 marks]*

Let $f(x) = x^2 - x$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f.
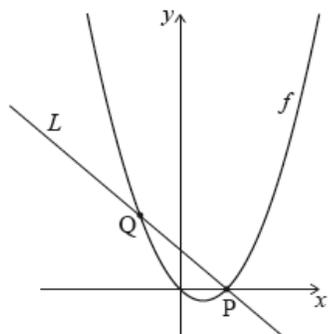
diagram not to scale

The graph of f crosses the x-axis at the origin and at the point $P(1, 0)$.

The line L intersects the graph of f at another point Q, as shown in the following diagram.
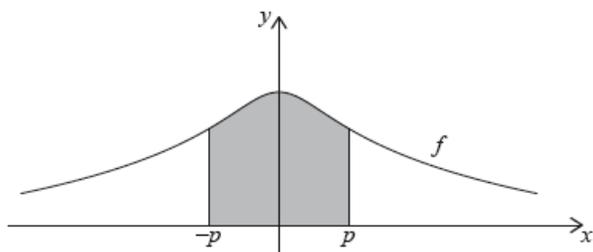
diagram not to scale

4. Find the area of the region enclosed by the graph of f and the line L.   *[6 marks]*

Let $f(x) = 6 - \ln(x^2 + 2)$, for $x \in \mathbb{R}$. The graph of f passes through the point $(p, 4)$, where $p > 0$.
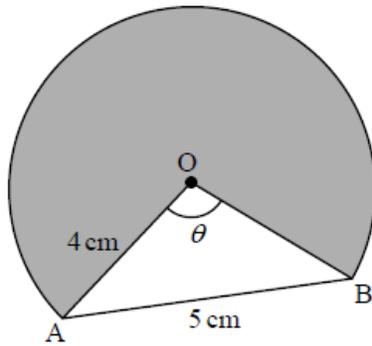
5a. Find the value of p.   *[2 marks]*

5b. The following diagram shows part of the graph of f.   *[3 marks]*

The region enclosed by the graph of f, the x-axis and the lines $x = -p$ and $x = p$ is rotated 360° about the x-axis. Find the volume of the solid formed.

The following diagram shows part of a circle with centre O and radius 4 cm.

Chord AB has a length of 5 cm and AÔB = θ.

6a. Find the value of θ, giving your answer in radians.   [3 marks]

6b. Find the area of the shaded region.   [3 marks]

The number of messages, M, that six randomly selected teenagers sent during the month of October is shown in the following table. The table also shows the time, T, that they spent talking on their phone during the same month.

Time spent talking on their phone (T minutes)	50	55	105	128	155	200
Number of messages (M)	358	340	740	731	800	992

The relationship between the variables can be modelled by the regression equation $M = aT + b$.

7a. Write down the value of a and of b.   [3 marks]

7b. Use your regression equation to predict the number of messages sent by [3 marks] a teenager that spent 154 minutes talking on their phone in October.

The price per kilogram of tomatoes, in euro, sold in various markets in a city is found to be normally distributed with a mean of 3.22 and a standard deviation of 0.84.

8a. Find the price that is two standard deviations above the mean price.   [1 mark]

8b. Find the probability that the price of a kilogram of tomatoes, chosen at   [2 marks] random, will be between 2.00 and 3.00 euro.

8c. To stimulate reasonable pricing, the city offers a free permit to the sellers whose price of a kilogram of tomatoes is in the lowest 20 %. *[2 marks]*

Find the highest price that a seller can charge and still receive a free permit.

The displacement, in centimetres, of a particle from an origin, O, at time t seconds, is given by $s(t) = t^2 \cos t + 2t \sin t$, $0 \le t \le 5$.

9a. Find the maximum distance of the particle from O. *[3 marks]*

9b. Find the acceleration of the particle at the instant it first changes direction. *[4 marks]*