HL2 Summer NO CALCULATOR Assignment *[49 marks]*

The functions f and g are defined such that $f\left(x\right) = \frac{x+3}{4}$ and $g\left(x\right) = 8x + 5$.

1a. Show that $\left(g \circ f\right)\left(x\right) = 2x + 11$. *[2 marks]*

1b. Given that $\left(g \circ f\right)^{-1}\left(a\right) = 4$, find the value of a. *[3 marks]*

Let $f\left(x\right) = \frac{16}{x}$. The line L is tangent to the graph of f at $x = 8$.

2a. Find the gradient of L. *[2 marks]*

L can be expressed in the form $\boldsymbol{r} = \begin{pmatrix} 8 \\ 2 \end{pmatrix} + t\boldsymbol{u}$.

2b. Find $\boldsymbol{u}$. *[2 marks]*

The direction vector of $y = x$ is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

2c. Find the acute angle between $y = x$ and L. *[5 marks]*
LEAVE ANSWER IN "ARCSIN" OR "ARCCOS" FORM.

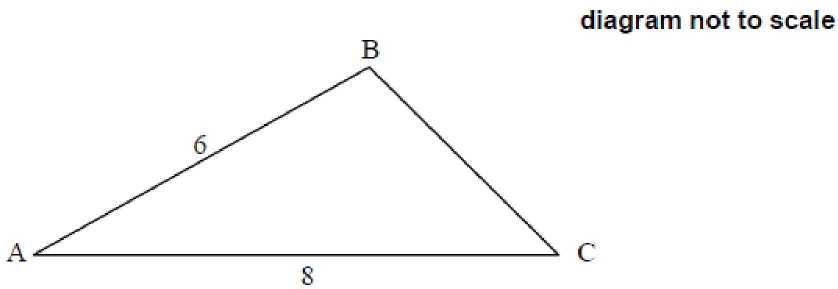
2d. Find $\left(f \circ f\right)\left(x\right)$. *[3 marks]*

2e. Hence, write down $f^{-1}\left(x\right)$. *[1 mark]*

Let $f(x) = (x - 5)^3$, for $x \in \mathbb{R}$.

3. Find $f^{-1}(x)$. *[3 marks]*

The following diagram shows triangle ABC, with *AB* = 6 and *AC* = 8.



4a. Given that $\cos \hat{A} = \frac{5}{6}$ find the value of $\sin \hat{A}$. *[3 marks]*

4b. Find the area of triangle ABC. *[2 marks]*

Let $f(x) = \frac{1}{3}x^3 + x^2 - 15x + 17$.

5a. Find $f'(x)$. *[2 marks]*

The graph of f has horizontal tangents at the points where $x = a$ and $x = b$, $a < b$.

5b. Find the value of a and the value of b. *[3 marks]*

5c. Sketch the graph of $y = f'(x)$. *[1 mark]*

5d. Hence explain why the graph of f has a local maximum point at $x = a$. *[1 mark]*

5e. Find $f''(b)$. *[3 marks]*

5f. Hence, use your answer to part (d)(i) to show that the graph of f has a local minimum point at $x = b$. *[1 mark]*

5g. The normal to the graph of f at $x = a$ and the tangent to the graph of f at $x = b$ intersect at the point (p, q) . *[5 marks]*

Find the value of p and the value of q.

Let $f(x) = x - 8$, $g(x) = x^4 - 3$ and $h(x) = f(g(x))$.

6a. Find $h(x)$. *[2 marks]*

6b. Let C be a point on the graph of h. The tangent to the graph of h at C is parallel to the graph of f. *[5 marks]*

Find the x-coordinate of C.